Perturbative solution to continuum generation in fiber gratings

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Abstract: We derive an approximate solution to the nonlinear Schrödinger equation which includes the effects of fiber gratings or other narrow-band spectral features. Our approach allows rapid estimation of grating enhancements from a single waveguide-only simulation.
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1. Introduction

It has recently been shown that when a continuum is generated in an optical fiber in the presence of a fiber grating, large spectral enhancements (>10x) can result near the grating [1]. These grating enhancements may have significant impact in frequency metrology, where they can be used to coherently amplify an arbitrary portion of a frequency comb in order to increase the power of the beat note that such a comb makes with another stabilized laser [2]. Other applications include enhanced 3rd harmonic generation [3] and novel multiwavelength sources [4]. These grating-induced enhancements were reproduced qualitatively in nonlinear Schrödinger equation (NLSE) simulations that included the grating transmission response in the NLSE. In particular, it was shown that the enhancement is on the long wavelength side of the grating band-gap, and could be as large as 20dB. While NLSE simulations provide a direct tool to compute the enhancement from a given grating, they provide little insight into the origin of the enhancements and can also be very time consuming due to the necessity for a very fine time/frequency grid to accommodate the disparate grating (~0.1THz) and continuum (>10THz) bandwidths.

In this paper we show that highly nonlinear propagation in the presence of a fiber grating or other narrowband spectral feature may be computed perturbatively. In our approach, the continuum acts as a source term for the grating induced E-field. The large dispersion near the grating bandgap generates the grating-induced dispersive wave, which can grow significantly as it is fed by the expanding continuum spectrum. When cross phase modulation (XPM) with the continuum is neglected, this growth can be computed from a z integral (along the fiber) over the unperturbed (no-grating) nonlinear propagation. The grating enhancement is then observed as spectral interference of the grating-induced and continuum E-fields. Our approach is valid near the grating bandgap, and requires 1) the dispersion length for frequencies near the Bragg wavelength is less than the nonlinear length for XPM between the continuum and grating waves, which results, in our treatment, from the small spectral width of the grating relative to the continuum; and 2) the grating does not significantly change the unperturbed solution away from the Bragg wavelength. Our method is valid for a wide range of nonlinear propagation and grating parameters and can be applied to strong Bragg gratings and their large spectral enhancements. Our simulations are in agreement with NLSE simulations using fs pulses and reproduce qualitatively the features observed in experiment.

2. Perturbative treatment of grating-continuum interaction

In the previous work [1,3], continuum generation within a fiber grating was modeled using a NLSE in which the dispersion operator was modified to include the effect of the grating:

$$\frac{\partial A(\omega)}{\partial z} = iD(\omega)A + i\delta\beta_{fbg}(\omega)A + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(\omega,\omega_1,\omega_2)A(\omega_1)A(\omega_2)A^*(\omega_1+\omega_2-\omega)d\omega_1 d\omega_2 \qquad (1)$$

Here A is the amplitude of the E-field, D is the fiber dispersion operator and third term is the Kerr nonlinearity [5]. The grating response is approximated with the additional complex propagation constant term $\delta\beta_{fbg}(\omega)$, derived from the grating response: $\delta\beta_{fbg} = \beta_{fiber+fbg} - \beta_{fiber}$, where $t_{fbg}(\omega) = \exp(i\beta_{fiber+fbg}(\omega)L)$ is the transmission of the grating computed from the coupled mode equations and L is the length of the grating.

In our approach we first compute the unperturbed nonlinear solution A_0 when $\delta\beta_{fbg}=0$. Then the effect of the grating enters as a perturbation: $A = A_0 + A_1$. When this is substituted into Eq. 1, A_1 satisfies:

$$\frac{\partial A_1(\omega)}{\partial z} = i\delta\beta_{fbg}A_0 + i(D+\delta\beta_{fbg})A_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(\omega,\omega_1,\omega_2)A_0(\omega_1)A_0(\omega_2)A_1^*(\omega_1+\omega_2-\omega)d\omega_1 d\omega_2 + \cdots \qquad (2)$$

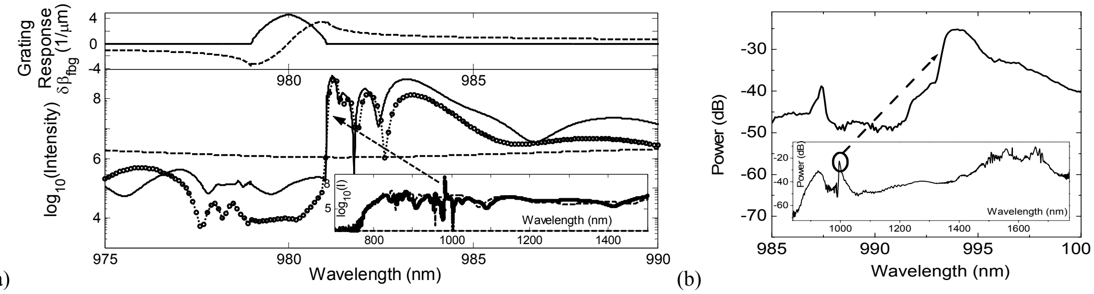
The key to our approximation is that we neglect nonlinear interaction between the grating-induced wave A_1 and the unperturbed continuum wave A_0, keeping only the first two terms on the right hand side. The A_1-A_0 XPM is small for two reasons: Firstly, the grating bandwidth is much smaller than the continuum bandwidth (~1/100), making one of the integrations in the Kerr term smaller by roughly the ratio of the grating and continuum bandwidths. This means that the effective nonlinear length for A_1-A_0 XPM is much longer than the grating dispersion length near the grating bandgap. This implies that our treatment is most accurate when $\delta\beta_{fbg}$ and hence the enhancement is large. Secondly, the grating field amplitude A_1 is small because the grating dispersion spreads the grating induced light A_1. In the time domain, these assumptions imply that A_1 has a small temporal overlap with A_0. With the XPM term eliminated, the solution to Eq. 2 may be written as an integral over A_0, and the approximate solution to Eq. 1 is then:

$$A(\omega) \cong A_0(\omega, L) + i\delta\beta_{fbg}\int_0^L A_0(\omega, z)e^{-i(D+\delta\beta_{fbg})(z-L)}dz \qquad (3)$$

Eq. 3 is exact if there is no Kerr nonlinearity, i.e., A_0 propagates linearly. The grating-induced peaks arise only when A_0 changes along z due to nonlinearities.

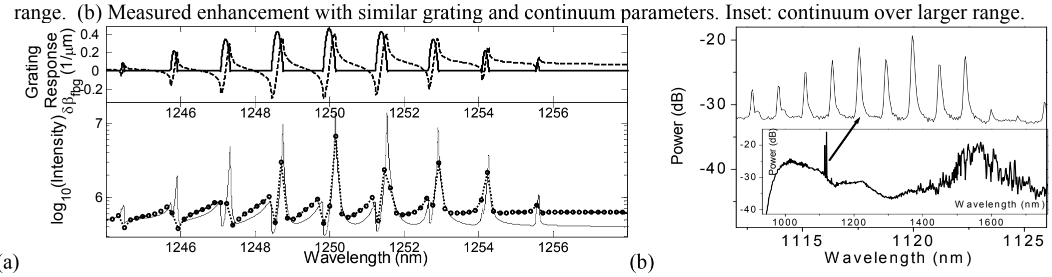
3. Comparison with NLSE simulations and experiment

We first considered a short 100% apodized grating: index modulation, δn=0.003, L=3cm, λ_{Bragg}=980nm. NLSE parameters: Gaussian pulse width 50fs, pulse energy 4nJ, fiber parameters as in [3]. Fig. 1 shows this comparison. As expected, the closest agreement is on the long wavelength side where the grating enhancement is largest. Fig. 1b shows a measured enhancement under similar conditions. The oscillations on the long wavelength side in Fig. 1a were observed in previous grating continuum measurements [3], but are not always present since additional nonlinear fiber and pulse to pulse variations can smooth them out as in Fig. 1b. We next chose a sampled grating similar to ref [4]: L=3cm; δn=0.002; Λ_{samp}=375µm; duty cycle 25%; λ_{Bragg}=1250nm. Fig. 2 shows that, within the simulation resolution, we again had good agreement with the full NLSE where the enhancement is largest.



(a) (b)

Fig. 1 Continuum generation in a fiber grating. (a) Top plot: Grating response. Solid: Im($\delta\beta_{fbg}$); dashed Re($\delta\beta_{fbg}$). Bottom Plot: Continuum intensity near Bragg resonance. Solid: Approximation (Eq. 3); connected circles: NLSE grating simulation (Eq. 1); dashed: Unperturbed solution ($\delta\beta_{fbg}$=0). Inset: continuum with and without grating on larger range. (b) Measured enhancement with similar grating and continuum parameters. Inset: continuum over larger range.



(a) (b)

Fig. 2 Sampled grating continuum. All lines as in Fig. 1. (a) Simulations. (b) Experiment with similar grating (ref [4]).

4. Conclusion

We have shown that the large spectral peaks observed near a Bragg resonance when continuum is generated in the Bragg grating can be described by treating the grating as a perturbation to the nonlinear propagation. The continuum generation in the fiber is computed once as a function of z in the fiber, and then combined with the grating transmission response to arrive at the grating-modified continuum without further solution of the NLSE. Thus, the response from any grating may be computed from a single computation of the nonlinear propagation, enabling rapid (seconds vs hours) simulation of the effect of fiber gratings or other narrow band resonances on continuum generation. Such grating enhancements have shown great potential to improve signal to noise ratio in frequency metrology applications using fiber continuum combs.

[1] P. S. Westbrook, *et al, Appl. Phys. Lett.* **85**, pp. 4600-4602, (2004).
[2] S. K. Kim, *et al,* CLEO Europe 2005 paper CP2-7.
[3] J.W. Nicholson, *et al,* CLEO 2005, paper CThU2.
[4] P. S. Westbrook, *et al*, OFC 2005 paper OThQ4.
[5] G. P. Agrawal, *Nonlinear Fiber Optics*, 2nd ed. (Academic, San Diego, 1995).