# Optimum intermediate fibers for reducing interconnection loss: exact solution

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We derive an exact analytical solution for a transmission line of $N$ single-mode intermediate optical fibers that minimize the interconnection loss between any two dissimilar fiber modes that are well described by that paraxial scalar wave equation. Our solution shows that $N$ optimum intermediate fibers reduce the original interconnection loss by a factor of least $1/(N+1)$ and that the total interconnection loss is only a function of $N$ and the original direct interconnection loss. Our solution is not restricted to axisymmetric fibers or mode fields and therefore could be useful for reducing the interconnection loss between microstructured optical fibers, between certain slab waveguides, or between fibers and optical sources or detectors.

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The rapidly expanding variety of optical fiber designs ensures that, despite the inherent difficulty, dissimilar fibers must frequently be interconnected. When interconnecting multimode fibers carrying a single-mode signal, such as double-clad fibers, energy lost from the operating mode at the interconnection point can couple into undesirable high-order guided modes, leading to gain competition and multipath interference. In high-power devices, such as optical fiber lasers and amplifiers, interconnection losses can lead to localized heating and associated reliability problems. Optical signals must also be transferred from chips into optical fibers and vice versa. Although there are alternate approaches for solving these problems by reducing the interconnection loss between two dissimilar optical fibers[1] or between a fiber and a chip, the best option is often the use of one or more intermediate fibers (also called bridge fibers or matching fibers[2]) that are inserted between the launching mode and the receiving mode (Fig. 1).

Although intermediate fibers necessitate more total interconnections, the total insertion loss can be substantially lower than the loss of a single direct interconnection. Previously there has been no accepted procedure for designing the optimum intermediate fiber(s) given two arbitrary dissimilar launching and receiving fiber modes. Here we derive an exact analytical solution for the global optimum intermediate fiber(s) for any two launching and receiving fiber modes obeying the paraxial scalar wave equation. We assume that the intermediate fiber is single mode and that it is long enough to ensure that radiation modes are fully attenuated, which is reasonable because radiation modes in single-mode fibers are typically attenuated by fiber coatings in less than 1 m. We also assume that reflections are negligible, which is reasonable because small amounts of diffusion in the axial direction during conventional fusion splicing is known to substantially suppress reflection between highly dissimilar fibers.[1] Naturally this global optimum yields lower loss than previous nonoptimal suggestions[2] and is much more general, since it is effective for any launching and receiving fiber designs, including those with non-Gaussian transverse electric field profiles. Our methodology is not restricted to axisymmetric fibers or mode field shapes and therefore could be suitable for interconnecting microstructured optical fibers or certain slab waveguides. When the launching and receiving mode field shapes exhibit weak wavelength dependence, our solution will be effective over a wide bandwidth.

When a launching fiber's signal mode, $E_{\mathrm{A}}$, encounters a receiving fiber's mode, $E_{\mathrm{B}}$ [Fig. 1(a)], the interconnection loss can be expressed in decibels as a function of the overlap integral

$$L_{\mathrm{A,B}}^{(0)} = -10 \log_{10}(\Omega_{\mathrm{A,B}}^2/\Omega_{\mathrm{A,A}}\Omega_{\mathrm{B,B}}), \tag{1}$$

where the superscript (0) indicates that there are no intermediate fibers present. The overlap integral $\Omega_{\alpha,\beta}$ in Eq. (1) is expressed as

$$\Omega_{\alpha,\beta} = \int E_\alpha E_\beta \mathrm{d}A, \tag{2}$$

where $E_\alpha$ and $E_\beta$ are the real spatial amplitudes of transversely polarized electric fields and the integration is performed over the entire cross-sectional area of the modes. Typically $E_{\mathrm{A}}$ and $E_{\mathrm{B}}$ are the $\mathrm{LP}_{01}$ fundamental modes of the launching and receiving fibers, but this derivation is equally valid for higher-

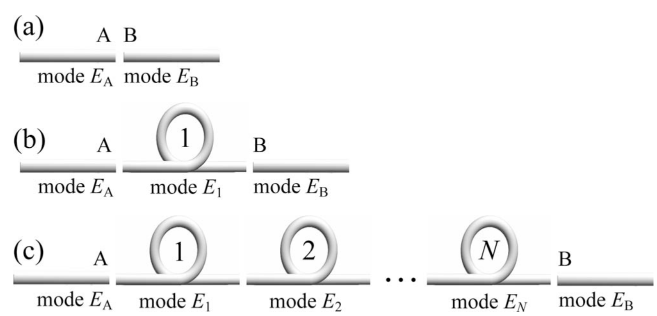


Fig. 1. (a) Direct interconnection between dissimilar fiber modes. (b) Single intermediate fiber ($N=1$) situated between launching and receiving fibers. (c) $N$ intermediate fibers.

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order guided modes of the source or receiving fiber as well. Note that the launching or receiving modes, $E_{\rm A}$ or $E_{\rm B}$, need not even be fiber modes at all, but instead could be optically conjugate to a pinhole source or detector.

Consider a transmission line consisting of $N$ single-mode fibers [Fig. 1(c)] connecting the launching and receiving fiber modes. Any optical energy that is not captured by the single guided mode of each intermediate fiber is assumed to be stripped out of the system. From Eq. (2), the fraction of power transmitted from launching mode $E_{\rm A}$ to receiving mode $E_{\rm B}$ is given by

$$T_{\rm A,B}^{(N)} = \frac{\Omega_{\rm A,1}^2 \Omega_{1,2}^2 \cdots \Omega_{N-1,N}^2 \Omega_{N,\rm B}^2}{\Omega_{\rm A,A}\Omega_{1,1}^2 \cdots \Omega_{N,N}^2 \Omega_{\rm B,B}}. \tag{3}$$

The total loss, $L_{\rm A,B}^{(N)}$, for $N$ intermediate fibers is

$$L_{\rm A,B}^{(N)} = -10 \log_{10} T_{\rm A,B}^{(N)}. \tag{4}$$

Our problem is to find the electric field distributions of the intermediate fibers $(E_1^{(N)}, E_2^{(N)}, \cdots E_N^{(N)})$ that maximize Eq. (3) given the launching and receiving fields ($E_{\rm A}$ and $E_{\rm B}$, respectively). We create a system of $N$ equations by zeroing the functional derivatives of Eq. (3) with respect to each intermediate fiber, $E_n$:

$$0 = \frac{{\rm d}T_{\rm A,B}^{(N)}}{{\rm d}E_n^{(N)}} = T_{\rm A,B}^{(N)} \left( \frac{2E_{n-1}^{(N)}}{\Omega_{n-1,n}} + \frac{2E_{n+1}^{(N)}}{\Omega_{n,n+1}} - 4E_n^{(N)} \right), \tag{5}$$

where the symbol $n-1$ is understood to mean $A$ when $n=1$ and $n+1$ is understood to mean $B$ when $n=N$.

Since the $\Omega$'s are constants, Eq. (5) shows that the mode field of each intermediate fiber is a linear sum of the preceding and subsequent mode field. This means that all the intermediate mode fields can be expressed as a linear sum of the launching and receiving mode fields:

$$E_n^{(N)} = a_n^{(N)} E_{\rm A} + b_n^{(N)} E_{\rm B}, \tag{6}$$

where $a_n^{(N)}$ and $b_n^{(N)}$ are coefficients. It is interesting to note that this equation implies that when $E_{\rm A}$ and $E_{\rm B}$ have Gaussian transverse profiles, the optimum intermediate fields do not. For convenience we now choose to normalize the launching and receiving mode fields, $E_{\rm A}$ and $E_{\rm B}$, such that

$$\Omega_{\alpha,\alpha} = 1. \tag{7}$$

When Eq. (6) is substituted into Eq. (5) and terms involving the independent functions $E_{\rm A}$ and $E_{\rm B}$ are grouped and zeroed we obtain [where the superscript ($N$) has been ommitted for brevity]

$$2a_n = \frac{a_{n-1}}{a_{n-1}a_n + b_{n-1}b_n + (a_{n-1}b_n + a_n b_{n-1})\Omega_{\rm A,B}} + \frac{a_{n+1}}{a_n a_{n+1} + b_n b_{n+1} + (a_n b_{n+1} + a_{n+1}b_n)\Omega_{\rm A,B}} \tag{8}$$

and another identical relation for $b$ where $a$ is exchanged everywhere with $b$. In both such equations the symbol $n-1$ is understood to mean $A$ when $n=1$ and $n+1$ is understood to mean $B$ when $n=N$. Remarkably, the entire system of $2N$ algebraic equations, and hence all the $a^{(N)}, b^{(N)}$, and the total optimized loss, all depend only on $N$ and $\Omega_{\rm A,B}$.

Since the coefficients are only functions of $\Omega_{\rm A,B}$ and $N$, but not direct functions of the spatial distribution of the launching and receiving modes, the coefficients will be the same when launching from $E_{\rm A}$ into $E_{\rm B}$ as when launching from $E_{\rm B}$ into $E_{\rm A}$. Therefore

$$a_n = b_{N-n+1}. \tag{9}$$

This means that when $N$ is even there are only $N$ independent algebraic equations to be solved and that when $N$ is odd there are $N+1$ independent equations, since the coefficients of the centrally situated intermediate fiber are equal [at $n=(N+1)/2$]:

$$a_{(N+1)/2} = b_{(N+1)/2}. \tag{10}$$

Since the system of algebraic nonlinear equations derived from Eqs. (8)–(10) can be numerically determined, the optimization problem is essentially solved. However, it is interesting to examine the solution in more detail for a few specific cases.

When $N=1$ we are considering a single optimized intermediate fiber, and Eq. (10) ensures that the electric field of this fiber's mode is an equal sum of the launching and receiving fields.

We find that

$$E_1^{(1)} = a_1^{(1)} E_{\rm A} + b_1^{(1)} E_{\rm B}, \tag{11}$$

where

$$a_1^{(1)} = b_1^{(1)} = 1/\sqrt{2 + 2\Omega_{\rm A,B}}. \tag{12}$$

So

$$L^{(1)} = -10 \log_{10}[(1 + \Omega_{\rm A,B})^2/4], \tag{13}$$

where $E_{\rm A}$ and $E_{\rm B}$ are normalized according to Eq. (7). Using Eqs. (1) and (13) one can show that $L^{(1)} < L^{(0)}/2$. This inequality approaches an equality as $L^{(0)} \to 0$. In most practical situations when $N=1$, the optimum intermediate fiber will reduce the total loss by very nearly a factor of two.

An optimal transfer of optical energy across the entire transmission line requires that the transmission over any subset also be optimal. Therefore, even when $N>1$, any optimal sequence of three fiber modes must obey Eqs. (11)–(13), where $E_{\rm A}$ is replaced by the first fiber mode and $E_{\rm B}$ is replaced by the third fiber mode. Optimal fields for large $N$ can be found and by repeatedly applying Eqs. (12) and (13) to neighboring fiber segments iteratively.

When $N=2$ we were unable to obtain a closed-form analytical expression for $a_n^{(2)}$ and $b_n^{(2)}$. Numerical or perturbative techniques may be the only option for explicitly solving this case. However, the optimal total loss, $L^{(2)}$, can be expressed in closed form as

$$L^{(2)} = -10\log_{10}\left\{\frac{1}{2} - \frac{1}{2}\cos\left[\frac{\arccos(1-2\Omega_{\mathrm{A,B}}^2)+2\pi}{3}\right]\right\}^3, \quad (14)$$

where $E_{\mathrm{A}}$ and $E_{\mathrm{B}}$ are normalized according to Eq. (7). Using Eqs. (1) and (14), one can show that $L^{(2)} < L^{(0)}/3$. This inequality approaches an equality as $L^{(0)} \to 0$.

When $N=2^m-1$, where $m$ is an integer, the optimization problem can be solved by recursion. Since there will be a centrally situated fiber in this case, from Eq. (10) we see that $a_{2(m-1)}^{(2^m-1)} = b_{2(m-1)}^{(2^m-1)}$ and, following Eq. (12),

$$a_{2(m-1)}^{(2^m-1)} = b_{2(m-1)}^{(2^m-1)} = 1/\sqrt{2+2\Omega_{\mathrm{A,B}}}. \quad (15)$$

The optimized field for the $2^{(m-1)}$th fiber, $E_{2(m-1)}^{(2^m-1)}$, is known and can now serve as the final receiving field for the first $2^{m-1}-1$ fibers, whereas it serves as the launching field for the final $2^{m-1}-1$ fibers in the sequence. This recursion enables us to obtain a closed-form solution for the case of $N=3(m=2)$:

$$a_1^{(3)} = b_3^{(3)} = \frac{\sqrt{2}\sqrt{\Omega_{\mathrm{A,B}}+1}+1}{\sqrt{2}\Omega_{\mathrm{A,B}}(\Omega_{\mathrm{A,B}}+1)^{3/2}+2\Omega_{\mathrm{A,B}}+\sqrt{2}\sqrt{\Omega_{\mathrm{A,B}}+1}+4}, \quad (16)$$

$$a_3^{(3)} = b_1^{(3)} = \frac{1}{\sqrt{2}\Omega_{\mathrm{A,B}}(\Omega_{\mathrm{A,B}}+1)^{3/2}+2\Omega_{\mathrm{A,B}}+\sqrt{2}\sqrt{\Omega_{\mathrm{A,B}}+1}+4}. \quad (17)$$

The optimal total loss, $L^{(3)}$, can be expressed as

$$L^{(3)} = -10\log_{10}\left(\frac{1}{2}+\sqrt{\frac{1+\Omega_{\mathrm{A,B}}}{8}}\right)^4. \quad (18)$$

Using Eqs. (1) and (18), one can show that $L^{(3)} < L^{(0)}/4$. This inequality approaches an equality as $L^{(0)} \to 0$.

We have found that the optimal total optical loss scales with the number of intermediate fibers as

$$L^{(N)} < L^{(0)}/(N+1), \quad (19)$$

(see Fig. 2). Inequality (19) follows from Eqs. (13), (14), and (18) for the cases of $N=1,2,3$ and has been numerically verified for cases when $N=2^m-1$ ($m \le 12$). It is reasonable to assume that it holds generally. When $N$ is large, our solution can be used to design a continuously varying transition between dissimilar launching and receiving fiber modes, but such a design ignores any energy that is lost to radiation modes and subsequently recoupled later on.

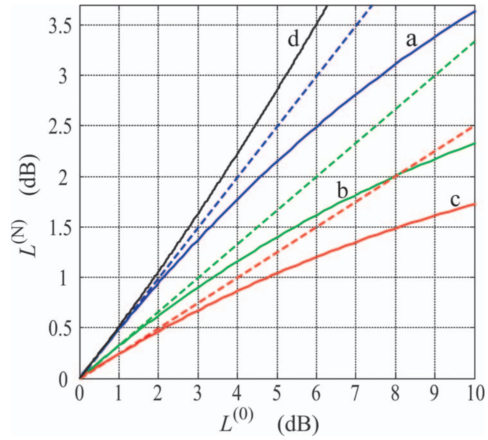


Fig. 2. Total optical loss using (curve a, solid blue) one optimal intermediate fiber, $L^{(1)}$; (curve b, solid green) two optimal intermediate fibers, $L^{(2)}$; and (curve c, solid red) three optimal intermediate fibers, $L^{(3)}$, as a function of original direct interconnection loss, $L^{(0)}$. The corresponding dashed lines show the asymptotes at small $\Omega_{\mathrm{A,B}}$, which are given by $1/(N+1)$. For comparison, the nonoptimal design suggested in Ref. 2 is shown (curved, solid black).

In our solution we have specified the optimal electric field distributions as a function of the number of intermediate fibers, $N$, and the launching and receiving field distributions, $E_{\mathrm{A}}$ and $E_{\mathrm{B}}$. Practical implementation of this solution requires the determination of the refractive index profiles that yield the desired intermediate fiber mode fields, which can be obtained by application of the paraxial scalar wave equation.[3]

In summary, we have derived an exact analytical solution in the form of a system of nonlinear algebraic equations to the problem of the optimum electric field(s) for reducing the loss between two dissimilar mode field profiles. Unlike previously suggested approximations, the exact solution derived here is quite general, as it applies to any fiber modes that are well described by the paraxial scalar wave equation, including nonaxisymmetric mode field shapes. This solution will be useful for designing intermediate fibers for interconnecting dissimilar optical fibers, including microstructured optical fibers.

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